BOOLEANISM AND BELIEF

Introduction

It seems obvious that believing a conjunction requires believing each conjunct. Whenever one believes that it is Monday and it is morning, one thereby believes that it is Monday and believes that it is morning. The principle, which I'll call *Distribution*, that it is metaphysically necessary that one believes a conjunction only if one believes the conjuncts of that conjunction, is widely regarded as a truism.[1]

It is also widely held that belief is not closed under entailment. Only idealized agents in distant possibilities believe all the consequences of what they believe. For agents like us, failing to draw an inference is commonplace. Call the thesis that it is metaphysically necessary that one believes any proposition entailed by a proposition one believes *Closure* and its negation *NoClosure*.

It is somewhat surprising that Distribution, NoClosure and the thesis that propositions form a Boolean algebra under the operations of conjunction and negation, which I'll call *Booleanism*, are mutually inconsistent—all the more so given the wide acceptance of Booleanism in both philosophy and linguistics.

Some philosophers, following Soames (1987), reject Booleanism in light of this problem. These authors often take Distribution and NoClosure to be premises in a powerful argument for the structured theory of propositions. A visible minority, most notably Stalnaker (1984), opt instead to accept Closure having been won over by the simplicity and strength of Booleanism and the purported obviousness of Distribution. This paper is a defense of the third way out, that of maintaining Booleanism while rejecting Distribution.

The fact that coarse grained views of propositions deliver implausible verdicts when combined with Distribution has convinced many to abandon such theories and seek out more fine grained alternatives. But if I am right, Distribution is false, and so the initial motivation to seek out more fine-grained alternatives is undercut. There may of course be other motivations to seek out such views. However since Booleanism constitutes an extremely simple and powerful theory of propositional granularity, absent some other devastating objection that does not rely on Distribution, there are good grounds for adopting Booleanism as a working hypothesis.

[1]Distribution is more often assumed than explicitly argued for. Here is a partial list of places in which the principle is either endorsed or presented favorably: Soames (1987), Dorr (2011), p. 957, Williamson (2000), p. 280 and Speaks (2006), p. 443.

Here is an overview of what is to come. §1 explains in more detail what Booleanism is. §2 explains why Booleanism is in tension with Distribution and NoClosure, and why the Boolean ought to respond to this tension by rejecting Distribution. §3 presents five arguments against Distribution. These arguments attempt to establish that the problem with Distribution is general, in the sense that it cannot be maintained without collateral damage even if one's background conception of propositions is a fine-grained one. I conclude in §4 with some discussion of why Distribution might seem true even if it is in fact false.

1. Booleanism

To formulate Booleanism I will suppose we are working in a language with a connective $\equiv$ to express the notion of propositional identity. Thus for two formulas φ and ψ, I will suppose that $\ulcorner \varphi \equiv \psi \urcorner$ is a formula. This formula can be read intuitively as $\ulcorner$The proposition that φ is the proposition that $\psi\urcorner$. Or if one prefers a more nominalistically acceptable gloss, $\ulcorner$For it to be the case that φ is for it to be the case that $\psi\urcorner$.[2]

With this notion in place, Booleanism can be formulated as the following schema:

Booleanism: $\varphi \equiv \psi$, whenever $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic.

The theory is so named because it is equivalent, given relatively uncontroversial logical assumptions, to the requirement that propositional identity satisfy the equations axiomatizing the class of Boolean algebras. Some of the more familiar formulas that follow from Booleanism are identifications such as $\varphi \equiv \neg\neg\varphi$ and $\neg(\varphi \vee \psi) \equiv (\neg\varphi \wedge \neg\psi)$. Some of the more controversial predictions of the view are, for instance, that $\varphi \equiv \varphi \wedge (\psi \vee \neg\psi)$ and $\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi)$.

Booleanism is a very *general* theory of propositions with many specific instances. For example, the now widely rejected theory of *extensionalism*, that $\varphi \equiv \psi$ whenever $\varphi \leftrightarrow \psi$, guarantees that Booleanism is true given the assumption that all propositional tautologies are true. A much more common example of the theory is *intensionalism*. Let $\Box$ be a monadic operator expressing metaphysical necessity. Intensionalism can then be formulated as a schema:

Intensionalism: $\Box(\varphi \leftrightarrow \psi) \rightarrow \varphi \equiv \psi$.

If Intensionalism holds, so does Booleanism, given relatively uncontroversial assumptions. For suppose that $\varphi \leftrightarrow \psi$ is a theorem of classical propositional logic. Since all such

[2]I will remain largely neutral on questions concerning the *ontology* of propositions, focusing solely on structural questions concerning propositions. The hypotheses put forward in this paper are consistent with a wide variety of ontological hypotheses concerning propositions. My own favored view is that talk of "propositions" is ultimately to be understood in terms of quantification into the position of a sentence and so not as involving the posit of any special class of abstract objects.

theorems are necessary, we can then infer that $\Box(\varphi \leftrightarrow \psi)$.[3] So given INTENSIONALISM, it follows that $\varphi \equiv \psi$.

One can maintain BOOLEANISM, however, while rejecting INTENSIONALISM. Some philosophers think that the temporal can vary freely of the modal, so that what is possible today will be impossible tomorrow.[4] Such theories seem to invalidate INTENSIONALISM, but are perfectly consistent with BOOLEANISM. To see this let A be the normal tense operator "it is always that the case that ..." and suppose that INTENSIONALISM holds. From the assumption that $\Box\varphi$, it follows that $\Box(\varphi \leftrightarrow \top)$. Thus from INTENSIONALISM, it follows that $\varphi \equiv \top$. Since A is normal, it follows that $A\top$. But since the proposition that φ *just is* the proposition that $\top$, it then follows, plausibly, that $\Box\varphi$.[5] Thus INTENSIONALISM predicts that $\Box\varphi \rightarrow A\varphi$. That is, INTENSIONALISM predicts that whatever is necessary is always true. That's inconsistent with the thesis that what is possible today may be impossible tomorrow. So INTENSIONALISM is inconsistent with the thesis that the temporal varies freely of the modal. But that thesis is not inconsistent with BOOLEANISM. For this thesis is consistent with an assumption that entails BOOLEANISM: that whenever two propositions are necessarily always equivalent, they are identical. That is, we can consistently suppose that $\Box A(\varphi \leftrightarrow \psi) \rightarrow (\varphi \equiv \psi)$ while also holding that $\Box\varphi \rightarrow A\varphi$ has untrue instances. And given our assumptions that both $\Box$ and A are normal modal operators, the former assumption suffices for BOOLEANISM.

Thus while BOOLEANISM provides a somewhat coarse grained view of propositions. It does not provide a view that is *required* to be as coarse grained as INTENSIONALISM. Indeed at the limit one could maintain a view on which, in a certain precise sense, the *only* propositional identities that hold are those that are predicted to hold from the Boolean axioms. A view like this would make some predictions that are *very fine grained*: for instance it would predict that the proposition that someone is a bachelor is distinct from the proposition someone is an unmarried male, since the assumption that all bachelors are unmarried males does not follow from logic alone.

BOOLEANISM does however predict something in the vicinity of INTENSIONALISM. Suppose we introduce an operator L into our language by the following stipulation:

$$L\varphi =_{df} \varphi \equiv \top$$

[3]For further discussion of the relation between theoremhood and necessity see Blanchette (2000).
[4]This sort of view is predicted by the logics of tense outlined in both Kaplan (1989) and Fine (1977) for instance.
[5]This argument can be made more rigorous by appealing to the principle of LEIBNIZ'S LAW discussed in the next section.

We can read $L\varphi$ as "The proposition that φ is logically necessary," or more simply as "It is logically necessary that φ". It will then be a consequence of BOOLEANISM that whenever it is logically necessary that two propositions are equivalent, they are identical. Formally:

BOOLEAN EQUIVALENCE: $L(\varphi \leftrightarrow \psi) \rightarrow \varphi \equiv \psi$

BOOLEAN EQUIVALENCE bears some formal analogues to INTENSIONALISM. Given plausible background assumptions, the operator L can be shown to be a normal modal operator whose logic is at least that of $\mathsf{S4}$. More importantly, Bacon (2018) argues that in a slightly stronger Boolean setting, L can be shown to be the *broadest necessity* operator, in a certain precise sense of 'necessity operator'. One conclusion that could be drawn from this is that L would then have to be metaphysical necessity since, as it is often conceived, metaphysical necessity *is* the broadest necessity.[6] This would then collapse the distinction between BOOLEANISM and INTENSIONALISM.

But there are some grounds for resisting the identification of L and $\Box$. The notion of metaphysical necessity was originally thought to be problematic because it was *discriminating*: for some qualitative feature F, it is metaphysically necessary that something is F while it is not metaphysically necessary that everything is F.[7] In other words, on the original conception of metaphysical necessity, it was supposed to validate certain important essentialist theses, such as, perhaps, the thesis that I couldn't have been a fried egg. It is not obvious, though, whether L should be thought to validate such theses. Indeed on one conception of how L should be thought to behave, the distinctive feature of it is precisely that it is *not* discriminating.[8]

Thus there are some grounds for distinguishing BOOLEANISM from INTENSIONALISM, taking the latter to be strictly stronger than the former. Importantly, a view on which metaphysical necessity is not L is nevertheless capable of doing all of the work INTENSIONALISM does, but without the more controversial assumption that propositions that are necessarily equivalent are identical. This means that we get a simple and strong theory of propositions, not requiring any sort of radical rethinking of standard possible worlds semantics, that is nevertheless capable of making more fine-grained distinctions.

2. THE CONFLATION ARGUMENT

One might have thought that the fact that BOOLEANISM is strictly weaker than INTENSIONALISM would allow the Boolean to escape some well known objections to the latter found in Soames (1987). In this section I argue that this is not so by showing how to generalize

[6]Dorr(2016) conditionally draws this conclusion. For dissenting views see Edgington (2004) and Nolan (2011).

[7]See Fine (1989) for a discussion of this point.

[8]See e.g., Bacon (forthcoming).

Soames' argument to an argument against Booleanism. Soames' original argument applied only to views that took a certain ontological stance on propositions: that they were sets of certain kinds. The argument presented here makes no such assumption and depends solely on theses concerning propositional fineness of grain. In my view, it is this argument that really gets to the heart of the matter by employing only those assumptions that are needed to generate the problem.

The argument deploys a dyadic sentential operator $\leq$ representing propositional entailment. So where φ and ψ are formulas, $\ulcorner \varphi \leq \psi \urcorner$ is a formula that intuitively says $\ulcorner$That φ entails that $\psi\urcorner$. In addition, we will use the monadic operator B to represent belief. So if φ is a formula, $\ulcorner B\varphi \urcorner$ is a formula that intuitively says $\ulcorner$One believes that $\varphi\urcorner$.

Consider the following three principles:

Leibniz's Law: $\varphi \equiv \psi \rightarrow (\chi \rightarrow \chi[\varphi/\psi])$
Distribution: $B(\varphi \wedge \psi) \rightarrow (B\varphi \wedge B\psi)$
Entailment: $\varphi \leq \psi \leftrightarrow L(\varphi \rightarrow \psi)$

Booleanism, together with these three principles, implies the following implausible psychological generalization.

Closure: $((\varphi \leq \psi) \wedge B\varphi) \rightarrow B\psi$

Here is why:

The Conflation Argument:

(1) $(\varphi \leq \psi) \wedge B\varphi$ (Premise)
(2) $L(\varphi \rightarrow \psi) \rightarrow (\varphi \equiv (\varphi \wedge \psi))$ (Booleanism, Leibniz's Law)[9]
(3) $(\varphi \leq \psi) \rightarrow (\varphi \equiv (\varphi \wedge \psi))$ (2, Entailment)
(4) $\varphi \equiv (\varphi \wedge \psi)$ (1, 3)
(5) $B(\varphi \wedge \psi)$ (1, 4, Leibniz's Law)
(6) $B\psi$ (5, Distribution)
(7) $((\varphi \leq \psi) \wedge B\varphi) \rightarrow B\psi$ (1, 6)

Since Closure looks to be an utterly unrealistic description of our own mental lives, one of the above four principles has to go. But, as I will explain shortly, there is good reason to think that *if* Booleanism is true, then Entailment is true.[10] This leaves the Boolean with only two options: reject Distribution or reject Leibniz's Law. The opponent of Booleanism presents this option as a reductio of the view. The conjunction of Distribution and

[9]Suppose $L(\varphi \rightarrow \psi)$. Then $\varphi \rightarrow \psi \equiv \top$. Since $(\varphi \wedge \psi) \leftrightarrow (\varphi \wedge (\varphi \rightarrow \psi))$ is a theorem of classical propositional logic, we can infer from Booleanism that $(\varphi \wedge \psi) \equiv (\varphi \wedge (\varphi \rightarrow \psi))$. And so since $\varphi \rightarrow \psi \equiv \top$ it follows from Leibniz's Law that $(\varphi \wedge \psi) \equiv (\varphi \wedge \top)$. But $\varphi \leftrightarrow (\varphi \wedge \top)$ is a theorem of classical propositional logic. So $\varphi \equiv (\varphi \wedge \top)$. Finally by Leibniz's Law, $\varphi \equiv (\varphi \wedge \psi)$.

[10]I will often engage in the sloppy behavior of applying truth to schemas. My meaning, which I hope is clear enough, is simply that every instance of the schema expresses a true proposition (ignoring here any issues concerning context sensitivity).

Leibniz's Law, they will say, is more plausible than the Booleanism. Thus if being a Boolean rationally requires rejecting either of these views, we ought to reject Booleanism.

One important strategy for resisting this argument is to deny Leibniz's Law. One might motivate this rejection as follows. Plausibly, despite the fact that Hesperus is Phosphorus, someone could believe that Hesperus is bright without believing that Phosphorus is bright. But if that is so, then it seems that we have a counterexample to the following schema:

Substitution: $a = b \to (\varphi \to \varphi[a/b])$

For if we substitute 'Hesperus' for a, 'Phosphorus' for b and 'one believes that Hesperus is bright' for φ, we obtain

Hesperus = Phosphorus →(one believes that Hesperus is bright→one believes that Phosphorus is bright)

But if Substitution has false instances, why think that Leibniz's Law has only true instances? The Boolean might thus respond by pointing to the failure of Substitution as grounds for rejecting Leibniz's Law.

Such a response risks a stale mate. Many of the main proponents of the Conflation Argument accept the view known as *Naïve Millianism*:[11]

Naïve Millianism $a = b \to (\varphi \equiv \varphi[a/b])$

On this view, if Hesperus is Phosphorus, then the proposition that Hesperus is bright *just is* the proposition that Phosphorus is bright; moreover, the proposition that one *believes* that Hesperus is bright just is the proposition that one believes that Phosphorus is bright. The problem is that Naïve Millianism and Leibniz's Law jointly entail Substitution. For suppose $a = b$ and φ. Then by Naïve Millianism, $\varphi \equiv \varphi[a/b]$. So by Leibniz's Law it follows that $\varphi[a/b]$.

On the other hand, if one rejects Naïve Millianism, it is not immediately clear why apparent failures of Substitution should lead to apparent failure of Leibniz's Law. For instance, perhaps while Hesperus is Phosphorus, one can believe that Hesperus is bright without believing that Phosphorus is bright; but that only leads to a counterexample to Leibniz's Law if we further assume that the proposition that Hesperus is bright is identical to the proposition that Phosphorus is bright. But those who reject Substitution will tend to reject this claim. Indeed, the failure of substitution is sometimes given as *grounds* for rejecting Naïve Millianism.

So while the rejection of Leibniz's Law is a *possible* response,, I'm inclined to think that if the Boolean really wants to rebut the argument, they should accept Leibniz's Law, at least for the sake of the argument, and show that despite this, the argument fails. The assumption of Leibniz's Law is a core part of most of our contemporary theories of propositions, and

[11] See e.g., Salmon (1986) and Soames (1987). For a more detailed overview see Saul (2017).

arguably follows from the assumption that 'the proposition that φ' is a theoretical term equivalent to its Ramsey sentence.

What about ENTAILMENT? Why should the Boolean accept this principle? Clearly the principle is not acceptable to someone who is *not* a Boolean. For suppose that $\varphi \leq \psi$ and $\varphi' \leq \psi'$. Then by ENTAILMENT, $\varphi \to \psi \equiv \top$ and $\varphi' \to \psi' \equiv \top$. And so since $\varphi \to \psi \equiv \varphi \to \psi$, it then follows that $\varphi \to \psi \equiv \varphi' \to \psi'$ by LEIBNIZ'S LAW.[12] But this sort of identification will only be acceptable to someone who is antecedently committed to a coarse grained view of propositions.

However once we suppose that BOOLEANISM is correct, ENTAILMENT presents itself as an attractive analysis of propositional entailment for a number of reasons.[13] The first reason is that it predicts that entailment plays a number of roles that, intuitively, it should. For instance, it follows from BOOLEANISM that entailment so defined is transitive and reflexive and interacts with the Boolean connectives in an expected way:

(i) $L(\varphi \to (\psi \wedge \chi)) \leftrightarrow (L(\varphi \to \psi) \wedge L(\varphi \to \chi))$
(ii) $L((\psi \vee \chi) \to \varphi) \leftrightarrow (L(\psi \to \varphi) \wedge L(\psi \to \chi))$
(iii) $L((\chi \wedge \varphi) \to \psi) \leftrightarrow L(\chi \to (\varphi \to \psi))$
(iv) $L((\varphi \wedge \neg\varphi) \to \psi)$ and $L(\psi \to (\varphi \vee \neg\varphi))$

It is also worth noting that if INTENSIONALISM is true, ENTAILMENT simply amounts to the thesis that one proposition entails another if the former necessitates the latter. But supposing we generalize from INTENSIONALISM to BOOLEANISM, it becomes natural to generalize our theory of entailment as well. It is a consequence of INTENSIONALISM that all necessary truths entail one another. By accepting ENTAILMENT, the Boolean who is not an Intensionalist thereby has a theory of propositional entailment with all the same formal features as INTENSIONALISM that nevertheless can recognize that some necessary truths may fail to entail others.

Thus ENTAILMENT provides a simple algebraic characterization of propositional entailment that bears many of the marks of propositional entailment. This seems to me to be grounds for the Boolean to accept ENTAILMENT. In fact it could be viewed as one of the motivations *for* BOOLEANISM that it is able to maintain ENTAILMENT: by going coarse grained, one is able to provide a simple, reductive account of propositional entailment in purely logical terms.

A further reason why this sort of simple account of entailment is desirable is that it can easily be "lifted" to provide a notion of *property entailment* and *relation entailment* that is

[12]The reflexivity of $\equiv$ follows from BOOLEANISM since $\varphi \leftrightarrow \varphi$ is a theorem of classical propositional logic. Without BOOLEANISM we need to add it in as an extra axiom governing $\equiv$.
[13]In what follows I will be taking LEIBNIZ'S LAW for granted.

free of any quantificational notions.[14] To extend the notion of entailment to relations, one can simply add to our language lambda terms and say that one relation R entails another S if

$$\lambda v_1 \ldots v_n L(R(v_1, \ldots, v_n) \to S(v_1, \ldots, v_n))$$

Dorr (2014) has emphasized the importance of such quantifier-free notions of property of entailment for current debates over quantifier variance. In terms of them, we can investigate the logic of the quantifiers using only identity and the truth functional connectives. Since there is plausibly no variance in meaning in the truth functional connectives, nor in identity, across communities whose patterns of reasoning with respect to these operations are the same, this gives us a stable way to investigate whether two communities who reason similarly with the quantifier expressions pick out the same quantifier by doing so.

Here is a final, more direct, argument for ENTAILMENT. Recall that given some further assumptions the operator L can be shown to be the *broadest* necessity operator. So if BOOLEANISM is true, then whenever $L(\varphi \to \psi)$, the conditional $\varphi \to \psi$ will be necessary, in *every* sense of necessity. But it's plausible that propositional entailment is represented by *some* sort of strict conditional. So if $\varphi \to \psi$ is necessary, in *every* sense of necessity, then $\varphi \leq \psi$ (i.e., that φ entails that ψ). Conversely, suppose that $\varphi \leq \psi$ and BOOLEANISM is true. Then since the proposition that φ entails the proposition that ψ, there shouldn't be any sense of possibility on which it comes out that $\varphi \wedge \neg\psi$ is possible. But if $\neg L \neg(\varphi \wedge \neg\psi)$, then there *is* a sense of possibility on which $\varphi \wedge \neg\psi$ is possible. So if BOOLEANISM is true, then $\varphi \leq \psi$ only if $\neg L \neg(\varphi \wedge \neg\psi)$. But we also know that if BOOLEANISM is true, then $L(\varphi \to \psi) \leftrightarrow \neg L \neg(\varphi \wedge \neg\psi)$. Thus if BOOLEANISM is true, ENTAILMENT is true.

A worry one could have is that this account of entailment also predicts that entailment is a *anti-symmetric* relation:

$$(L(\varphi \to \psi) \wedge L(\psi \to \varphi)) \to \varphi \equiv \psi$$

Perhaps some will insist that is desirable to have a notion of entailment that allows for distinct propositions to entail one another. But the anti-symmetry of propositional entailment is usually taken to be one of the distinguishing features of coarse grained views of propositions in general and so shouldn't be particular worrying to the Boolean. Additionally, the Boolean can maintain that there are *weaker* entailment relations on propositions that are not symmetric. For instance, if INTENSIONALISM is false, then the relation $\Box(\varphi \to \psi)$ may still represent a theoretically interesting entailment-like relation (for instance, such a relation may be important for stating various supervenience hypotheses in the philosophy of

[14] In order for the proposed extension to be plausible one should also extend BOOLEANISM to cover properties and relations. See Dorr (2016) for a formulation of this view.

mind and other fields). The relation $L(\varphi \to \psi)$, however, is still a distinguished one for the Boolean and so seems to me to deserve the name of propositional entailment.

The Boolean thus has reason to accept ENTAILMENT. And while one can perhaps motivate rejecting LEIBNIZ'S LAW, doing so is not particularly dialectically effective in the present context since it is often a core commitment of the proponent of the Conflation Argument. This leaves the Boolean with only one option: reject DISTRIBUTION.

For many, DISTRIBUTION has seemed an obvious truth: a premise not even in need of defense. There would be just something utterly bizarre with admitting that one did not believe that φ while also admitting that one believed that φ *and* ψ. However, I have come to think that DISTRIBUTION is false, and so that the Boolean should respond to this argument by rejecting it. In the remainder of this paper, I will provide five arguments against DISTRIBUTION. The first two arguments concern specific cases in which I think the principle may fail. The last three arguments provide more indirect theoretical arguments for its falsity. Taken together, these arguments seem to me to provide ample evidence that DISTRIBUTION is false.

3. FIVE ARGUMENTS AGAINST DISTRIBUTION

The first three arguments I will give will depend on the following principle connecting sentential acceptance and the propositions one believes:

> DISQUOTATION: If S expresses in L the proposition that P, and one understands and sincerely accepts S, then one believes that P.

The truth, or at least approximate truth, of DISQUOTATION seems to me to be a foundational presupposition of our practice of reporting both the beliefs of others and our own beliefs. If it turned out to be false, our grounds for attributing specific beliefs in others would be in jeopardy. But even if one ultimately rejects it, given that it too is a widely accepted principle, it is still of independent interest that it can be used to generate counterexamples to DISTRIBUTION.[15]

3.1. **The First Argument.** Consider the following scenario.[16]

> THE LOGICIANS
>
> Alice encounters a community of logicians who use the symbol '$\triangle$' to express conjunction. Thus when a member of this community sincerely and assertively utters $\ulcorner \varphi \triangle \psi \urcorner$ they assert the proposition that we would by sincerely and assertively uttering $\ulcorner \varphi \land \psi \urcorner$. One day Alice asks one of the logicians what they mean by '$\triangle$'; the logician somewhat unhelpfully responds, "$\triangle$ is the most

[15]This principle is proposed in Kripke (1979). For a defense see Speaks (2010).

[16]Thanks to REMOVED FOR BLIND REVIEW.

natural relation between propositions that is such that, for any propositions p and q the following material bi-conditional holds:

$$p \triangle q \leftrightarrow \big(\neg(\neg p \vee \neg q) \wedge (p \to (q \to \top)) \to (p \to q) \to (p \to \top)\big)$$

Now, having grasp of all the terms that occur on the right hand side of this bi-conditional, and also having some grasp on what the naturalness of a relation consists in, Alice thereby gains some grasp of what '$\triangle$' means.

After spending some time in this community, occasionally uttering sentences of the form '$\varphi \triangle \psi$' when everyone else seems to be doing so as well, a logican, Claire, whom Alice views as a great authority says to Alice, "Grass is green $\triangle$ the past is finite." Since Alice has some grasp of what $\triangle$ means, and trusts the testimony of this logician, she thereby comes to sincerely accept "Grass is green $\triangle$ the past is finite" and thereby *believe* that grass is green $\triangle$ the past is finite. Despite this fact, she does *not* believe that the past is finite.

This sort of case seems to me to be a possible one. And given its possibility, it seems to me to show that DISTRIBUTION is false. For Alice comes to believe that grass is green and the past is finite by way of sincere acceptance of 'grass is green $\triangle$ the past is finite', maintaining her belief throughout that the past is *not* finite.

Here is a more rigorous presentation of the argument. By the description of the case we have the following three premises:[17]

(L1) '(grass is green $\triangle$ the past is finite)' expresses the proposition that grass is green and the past is finite.

(L2) Alice understands and sincerely accepts '(grass is green $\triangle$ the past is finite)'.

(L3) It is not the case that Alice believes that the past is finite.

But then (L1) and (L2), together with DISQUOTATION entail (L4):

(L4) Alices believes that grass is green and the the past is finite.

But (L4) taken together with (L3) provides a counterexample to DISTRIBUTION.

The premises (L1) and (L3) seem to me to follow simply from the description of the case. The more controversial premise is probably (L2). One way to motivate rejecting this premise is to deny that Alice really does understand '(grass is green $\triangle$ the past is finite)'. But it wouldn't do to support this position to merely cite that Alice is unaware of some of the logical consequences of S. In general most people understand sentences of the languages they speak without being able to assess what those sentences logically entail. It also wouldn't do to

[17] In this argument and the next I am going to behave in the sloppy behavior of pretending that the cases described are *actually* true, as opposed to presenting merely hypothetical cases. Nothing of substance turns on this, however.

merely cite as evidence for her not understanding this sentence that she does not know that it expresses the same proposition as 'grass is green and the past is finite'. The Millian, for instance, grants that 'Hesperus is bright' expresses the same proposition that 'Phosphorus is bright' despite the fact that many speakers who understand both sentences do not know this fact. Indeed this assumption plays a crucial role in their account the pragmatics of belief reports more broadly (and in fact plays a central role in Soames' (1987) argument against coarse grained views of propositions).

Perhaps one could argue that Alice does not understand the sentence in question since she is not disposed to reason in accordance with its introduction and eliminations rules: she is not disposed to accept '$\varphi \triangle \psi$' only if she accepts φ and ψ. But this places too strong of a constraint on understanding. First, reflection on the preface paradox arguably shows that *we* might not be disposed to reason in accordance with the introduction rules for $\wedge$, even provided that we are disposed to reason in accordance with its elimination rules. Second, the introduction and elimination rules for *pejorative* expressions arguably involves an introduction rule from a basic descriptive premise, and an elimination rule to some sort of insulting normative conclusion: thus in general one is disposed to reason in accordance with the introduction and elimination rules for pejorative expressions only if one is a bigot. But this certainly doesn't mean that it is only the bigots who understand pejorative expressions. And finally, in any linguistic community involving some philosophers one is likely to find *disagreement* over what the introduction and elimination rules are for a variety of logical expressions. Some people think that "Sherlock Holmes is a fictional character" implies "There are fictional characters" while others disagree. This seems to me to be a genuine disagreement, not reflecting some sort of lack of understanding of the quantificational locution "there are." If that's right, then in general one can understand a logical expression without being disposed to reason in accordance with its introduction and elimination rules.

For these reasons I think we should accept (L2). Thus, I think we should reject DISTRIBUTION. Now I imagine some might not find this cases too convincing, refusing to believe Alice really *could* understand the sentence in question without realizing it merely expressed a conjunction. But instead of dwelling on this, let's look at the next example: while it involves a more controversial assumption, the sort of "failure to realize one accepts a conjunction" that occurs in this this case seems to me more apparent.

3.2. **The Second Argument.**

The second argument I want to give depends on the following controversial assumption about propositional fineness of grain:

INVOLUTION: $\neg\neg\varphi \equiv \varphi$

Of course certain very fine grained accounts of propositions will deny this principle. On these views the proposition that φ cannot be the result of applying some operator to the negation of the proposition that φ. But the fact that these views entail that INVOLUTION is false seems to me more of an objection to those views than to INVOLUTION.[18] But even if someone disagrees with this claim, the fact that we can construct an argument against DISTRIBUTION *merely* by assuming INVOLUTION is significant, since this latter principle is maintained by many of the *consistent* fine-grained views of propositions currently on offer.[19] If we can generated counterexamples to DISTRIBUTION merely by assuming INVOLUTION, this helps to make clear how wide ranging the consequences of the former principle is.

Supposing then that INVOLUTION is correct, consider the following scenario.

> CONDITIONAL TWIN EARTH
> Suppose that there is a Twin Earth with a community much like our own in certain respects, except that on Twin Earth, people speak Twenglish *instead* of English, where Twenglish is language that differs at most from English in that in Twenglish, indicative conditionals express the material conditional. In particular, suppose that every instance of the following schema holds (ignoring context sensitivity for present purposes):
>
> ‘If φ then ψ’ expresses in Twenglish that $\neg(\varphi \wedge \neg\psi)$.
>
> Now suppose a speaker, Alice, is transported from Earth to Twin Earth without her knowing. Because of the overwhelming similarity between these communities, she goes on to live there, unaware that the switch took place for 20 years. One day her friend tells her, “If there is an earthquake tomorrow, your house will collapse.” Alice indignantly rejects his claim, “What? That’s not true. My house is solid. It’s not the case that if there is an earthquake tomorrow, my house will collapse.” Her friend of course reacts with surprise, since, by his lights, she has just predicted that there *will* be a major earthquake tomorrow.

The above case seems to me to be one in which DISTRIBUTION fails. The description of the case supports the truth of the following three premises:

(C1) Alice is a competent speaker of Twenglish, who understands and sincerely accepts ‘It is not the case that if there will be an earthquake tomorrow, my house will collapse’.

[18] For an argument that we ought to accept INVOLUTION see Ramsey 1927. For a formulation of this argument more in line with present notation see Dorr (2016).

[19] The structured view defended by Soames (1987) can arguably be shown inconsistent by an argument from Russell (1903, Appendix B). While this is an old problem, only in the past couple of years have philosophers gotten around to actually proposing fine-grained views of propositions that could get around it (though it may be that Frege’s view of senses avoids the problem, see Goodman (2017) for a defense).For some of this work Dorr(2016), Goodman(2019).

(C2) It is not the case that Alice believes that there will be an earthquake tomorrow.

(C3) ‘It is not the case that if there will be an earthquake tomorrow, my house will collapse’ expresses in Twenglish that $\neg\neg$(there will be an earthquake tomorrow $\wedge\neg$my house will collapse).

From (C1), (C3) and Disquotation we have

(C4) Alices believes that $\neg\neg$(there will be an earthquake tomorrow $\wedge\neg$my house will collapse).

And so from (C4), Involution and Leibniz’s Law we can infer:

(C5) Alice believes that (there will be an earthquake tomorrow $\wedge\neg$my house will collapse).

But (C5) and (C2) jointly provide a counterexample to Distribution.

As before, one could challenge my claim that Alice is a competent speaker of the language in question. But this seems even more implausible in this case. Alice is assumed to have lived on Twin Earth for twenty years. The standard judgment about Twin Earth cases is that at some point, Alice begins speaking their language, rather than the language she started out speaking when she arrived.

To further support this intuition, notice that it *might* actually be that the indicative conditional means the material conditional *in English*. This thesis has been ably defended by many philosophers. But if that is true, we don’t even need to suppose Alice has been transported to a different community. Normal English speakers probably *would* deny “If there is an earthquake tomorrow, your house will collapse” on the grounds that their house is well built. Those who defend the material interpretation of indicatives often explain away this behavior either in terms of Gricean pragmatics or in terms of our reliance on imperfect heuristics for evaluating conditionals.

The assumption of Involution could instead be replaced by the assumption that belief is closed under the rule of double negation elimination:

$\neg$-Elim: $B\neg\neg\varphi \to B\varphi$.

The argument then shows that *if* belief is closed under double negation elimination, it is *not* closed under “conjunction elimination.” That is, if *neg*-Elim is true, Distribution is false. But without Involution, $\neg$-Elim doesn’t look particularly motivated to me. There are, after all, some logicians who think that the formula $\ulcorner\neg\neg\varphi\urcorner$ does not logically imply φ. If the proposition that $\neg\neg\varphi$ is then not the proposition that φ, it is natural to describe their state of mind by attributing to them the belief that the proposition that $\neg\neg\varphi$ does not entail the proposition that φ. But if the proposition that $\neg\neg\varphi$ just is the proposition that φ, there is less motivation to describe their beliefs this way since it involves attributing to them straightforwardly inconsistent beliefs.

Both of the above cases involve a speaker coming to believe that φ and ψ, by accepting a misleading *guise* of the proposition that φ and ψ. In the first case Alice comes to believe that φ and ψ via the guise '$\varphi \triangle \psi$'; in the latter case, Alice comes to believe that φ and ψ via the guise 'it is not the case that if φ, ψ'. It is worth noting that the *kinds* of failures of Distribution that the Boolean accepts are exactly of this sort. For instance suppose I come to believe that Clark Kent is a journalist and Superman flies by accepting the sentence 'Clark Kent is a journalist and Superman flies'. Plausibly though, the proposition that that Clark Kent is a journalist and Superman flies entails the proposition that someone is both a journalist and flies. And so if Booleanism and Entailment are true, then the proposition that Clark Kent is a journalist and Superman flies is identical to the proposition that Clark Kent is a journalist and Superman flies and some journalist flies. So one must thereby come to believe that Clark Kent is a journalist and Superman flies and some journalist flies under the misleading guise of the sentence 'Clark Kent is a journalist and Superman flies'. And one can come to believe what 'Clark Kent is a journalist and Superman flies' expresses without thereby believing that some journalist flies. So Distribution has many false instances according to the Boolean. But these false instances are of the same kind as the two examples above: the Boolean thinks that the phenomenon of misleading guises is just much more widespread than more fine grained accounts of propositions allow for.

I conclude that both of these cases provide counterexamples to Distribution. Moreover, these counterexamples are predicted by the exact same kinds of factors that should lead the Boolean to reject Distribution. No theory of propositions can rule out there being cases in which one misleadingly represents some proposition. In general there is simply no way to make absolutely apparent to someone the nature of the content represented by some representation that represents that content.

3.3. **The Third Argument.** The third argument that looks to potential counterexamples of a similar sort to the last one, but seems to me to have a better chance of providing counterexamples that *actually* occur, as opposed to merely possibly occur. Consider the following principle:

$\exists$-Distribution $B\exists x(\varphi \wedge \psi) \rightarrow (B\exists x\varphi \wedge B\exists x\psi)$

According to this principle, if one believes that something is green and square, one believes that something is green and one believes that something is square. I will first argue that this principle has counterexamples. I will then argue that *if* this principle has counterexamples, so does Distribution.

The argument against $\exists$-Distribution relies on a controversial principle somewhat analogous to Involution.

Duality: $\neg\forall x(\varphi \rightarrow \psi) \equiv \exists x(\varphi \wedge \psi)$

Suppose that 'All F are Gs' expresses the proposition that $\forall x(Fx \to Gx)$. Suppose that 'some Fs are Gs' expresses the proposition that $\exists x(Fx \wedge Gx)$. Then according to Duality, 'not all Fs are Gs' expresses the proposition that that some Fs are not G (i.e., $\exists x(Fx \wedge \neg Gx)$). Now the assumption that the proposition that not all Fs are G is the proposition that some F is not G strikes me as a pretty minimal one. It is consistent with a variety of fine-grained views of propositions, though will likely be rejected by those who endorse the most fine-grained conceptions of propositions according to which any difference in logical form makes for difference in which propositions is expressed. There is also metaphysical reason to take it seriously. For suppose we wanted some fundamental description of the world. It would be unfortunate if it turned out that we had to decide between the statements 'Some F are G' and 'not all Fs are G' in this description. The principle of Duality says that either sentence will do because they pick out the very same proposition.

With these assumptions in place, consider the following example. Suppose Alice holds the mistaken belief that 'All Fs are G' logically entails 'Some F is not G'. Alice is also a presentist, and so believes that there are not any dinosaurs. On the basis of these two beliefs, she thereby sincerely rejects 'All dinosaurs are dinosaurs'. That is to say, she sincerely *accepts*, 'Not all dinosaurs are dinosaurs'. So by Disquotation:

(D1) $B(\neg\forall x(Dx \to Dx))$

And so from Duality and Leibniz's Law, we can infer:

(D2) $B\exists x(Dx \wedge \neg Dx)$

But by the description of the case, we have:

(D3) $\neg B\exists x Dx$

And so (D2) and (D3) jointly constitute a counterexample to $\exists$-Distribution.

The argument against $\exists$-Distribution could be extended to an argument against Distribution if we assumed the following principle:

Unrestricted Exportation: $(\exists x\varphi \wedge B\exists x\varphi) \leftrightarrow \exists x B\varphi$

Suppose, for example, that presentism is false and that there are dinosaurs. Then by two applications of Unrestricted Exportation the above example gives:

$$\exists x B(Dx \wedge \neg Dx) \wedge \neg\exists x B Dx$$

That's inconsistent with the following instance of Distribution:

$$\forall x(B(Dx \wedge \neg Dx) \to (BDx \wedge B\neg Dx))$$

However the thesis Unrestricted Exportation is not widely held. There a clear, intuitive difference between believing that there are spies and believing of some x that x is a spy. That being said, there *are* many contexts in which the existential quantifier seems to be exportable over belief.[20] And so given that there are such cases, it is not unreasonable to expect that at least *one* of them will provide a false instance of DISTRIBUTION.

Now one response that is more salient in this case is the following. Suppose that one mistakenly believes that there are no lions and mistakenly believes "All lions are lions" entails "there are lions." Then we they come to believe that not all lions are lions, they come to believe that some lion is not a lion. Now one could then point at a given lion and say "they believe that that is both a lion and not a lion" to point out the contradiction in their beliefs. But anyone who said this would presumably also say "they believe that that is a lion." In other words, instead of saying these sorts of cases provide counterexample to DISTRIBUTION, they might just say they show that the person in question acquires more inconsistent beliefs that it may have initially seemed.

I think this argument exploits the fact that it is hard to attribute a belief to someone using a conjunctive sentence without at the same time, attributing the belief in both conjuncts. Later, I will provide an explanation for why this might be so. But for the purposes of responding to think objection, it suffices to point out that the subject will sincerely deny believing that there are lions. Moreover, in this case, there is no *other* sentence we can point to as a misleading guise of the proposition that there are lions that she sincerely accepts. So there is no obvious reason, other than the initial plausibility of DISTRIBUTION, to say that she does believe that there are lions.

3.4. **The Fourth Argument.** The fourth argument I want to present is due, in its essentials, to Alexander Pruss.[21] The argument takes off from well known work on the *conjunctive fallacy*, in which some agent assigns a higher probability to a conjunction than one of its conjuncts. One presentation of the argument relies on the *Lockean Thesis*, on a strong reading of it, which says that to believe that something is so is to be sufficiently confident that it is so. Let $Cr(\varphi) = x$ express that one's credence in the proposition that φ is x. One way of interpreting the strong reading of the *Lockean Thesis* is as asserting that there is some threshold r such that to believe that φ is for one's credence in the proposition that φ to not fall below r:[22]

LOCKEAN THESIS: $B\varphi \equiv (Cr(\varphi) \geq r)$

The LOCKEAN THESIS, as I have formulated it, is particularly strong. It asserts that believing that φ *just is* having a sufficient high credence in the proposition that φ.[23] A structured proposition theorists might reject this on the grounds that the proposition that

[20] For discussions of this point see .

[21] See http://alexanderpruss.blogspot.com/2011/10/does-belief-distribute-over-conjunction.html. Pruss provides several further arguments there, none of which strike me as particularly convincing.

[22] For a defense see Leitgeb (2017). For criticisms see Buchak (2014).

[23] For a more detailed overview on the possible relations between belief and credence see Jackson (2020).

$B\varphi$ has a different structure than the proposition that $Cr(\varphi) \geq r$. However the argument that follows actually only depends on the weaker thesis that believing that φ is necessarily equivalent to having a credence that doesn't fall below some threshold, which is perfectly compatible with the structured propositions.

Now consider the following argument, where $\Diamond$ represents metaphysical possibility:

(P1) $\Diamond Cr(\varphi \wedge \psi) > C(\varphi)$
(P2) $\Diamond(Cr(\varphi \wedge \psi) > C(\varphi)) \rightarrow \Diamond(Cr(\varphi \wedge \psi) \geq r) \wedge (Cr(\varphi) < r))$
(P3) $\Box(Cr(\varphi \wedge \psi) \geq r) \wedge (Cr(\varphi) < r)) \rightarrow (B(\varphi \wedge \psi)) \wedge \neg B\varphi)$
(C) $\Diamond B(\varphi \wedge \psi) \wedge \neg B\varphi)$.

(P1) is justified on empirical grounds. The most famous example is due to Tversky and Kahneman (1983). They provide the following scenario:

> Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Which is more probable?
>
> (a) Linda is a bank teller.
> (b) Linda is a bank teller and is active in the feminist movement.

If once credences conform to the laws of probability, one shouldn't assign a higher credence to (b) than (a) since (b) entails (a). Yet we find that many people are disposed to judge (b) more probable that (a). Given that this actually occurs, certainly it possibly does. So (P1) should be true.

(P3) follows from the Lockean Thesis: necessarily if one's credence in $\varphi \wedge \psi$ is above the threshold and one's credence in φ is below the threshold, then one believes that $\varphi \wedge \psi$ but one does not believe φ.

What about (P2)? Suppose that it is possible that $Cr(\varphi \wedge \psi) > Cr(\varphi)$. Then it is possible that $Cr(\varphi \wedge \psi) - Cr(\varphi)$ is some positive number s. Now if $Cr(\varphi \wedge \psi) < r$, it is difficult to see why it shouldn't be possible to increase one's confidence in $\varphi \wedge \psi$ so that $Cr(\varphi \wedge \psi) = r$, while at the same time keeping $Cr(\varphi \wedge \psi) - Cr(\varphi) = s$. Similarly if $Cr(\varphi \wedge \psi) \geq r$, it is difficult to see why it shouldn't be possible to either keep one's credences the same or lower them, so that $Cr(\varphi \wedge \psi) = r$ while still ensuring that $Cr(\varphi \wedge \psi) - Cr(\varphi) = s$. In both cases one guarantees that $Cr(\varphi \wedge \psi)$ is at the threshold while $Cr(\varphi)$ is below it. In other words, the kinds of cases we see in the conjunctive fallacy to not seem to depend in any robust sense what one's actual credences in $\varphi \wedge \psi$ and φ are. What matters is the relationship between them.

One might be inclined to respond as follows, "In the conjunction fallacy, there is a small difference in credence, but certainly that difference couldn't amount to the difference between outright belief and lack of outright belief. That's implausible." But this objection is not to (P3) but rather to the LOCKEAN THESIS. It is not my intention here to offer any sort of defense of that principle. But given its popularity, it is still worth noting that it leads to conflicts with DISTRIBUTION. There is also a somewhat straightforward response here. Whenever one's credences are just above the threshold, one presumably cannot safely believe, and so cannot know, that one's credences are above the threshold. In that case, while there may be failures of DISTRIBUTION predicted by the LOCKEAN THESIS, they are not failures any subject is an position to *know* occur.

So supposing that the LOCKEAN THESIS is true, possibly DISTRIBUTION has false instances. But the LOCKEAN THESIS as I have formulated assumed a controversial view on the relationship between credence and belief. Jackson and Moon (2020) defend the view that belief is prior to credence. Does the LOCKEAN THESIS contradict this? Suppose the following is true:

BELIEF FIRST:$(Cr(\varphi) = x) \equiv B(Pr(\varphi) = x)$

This view is not obviously *inconsistent* with the LOCKEAN THESIS; though it does lead to some rather strange results. In particular it predicts something of a regress:

$$B\varphi \equiv B(Pr(\varphi) > r) \equiv BPr(Pr(\varphi) > r) > r \equiv \dots$$

But this regress isn't clearly a vicious one. For instance it might be that all a subject needs to do to fix the their beliefs in higher-order probabilities is to be such that their beliefs in lower probabilities are fixed. That being said it is not necessarily the most desirable consequence, and so a belief first type view could be put forward as a response to this argument, though the exact steps in that argument would have to be worked out more fully. I want to now move on to my fifth argument against DISTRIBUTION.

3.5. **The Fifth Argument.** The fifth and final argument against DISTRIBUTION takes off from the observation that in addition to using ascriptions like "Alice believes that grass is green and snow is white" to report conjunctive beliefs, we also use ascriptions like "Alice believes the conjunction of Logicism and Platonism" to reports conjunctive beliefs. Importantly, the 'and' as it occurs in "Logicism and Platonism" is not a sentential operator, but rather takes two terms to form a *plural term*. Thus the phrase "the conjunction of ..." combines with a plural term to form a non-plural term naming the conjunction of the propositions named by the plural term. This sort of talk seems to me to be in perfectly good order.

Plausibly, the state that Alice is reported to be in when one says "Alices believes the conjunction of Logicism and Platonism" is the same as the state that Alice is reported to be in when one says "Alices believes that math reduces to logic and there are abstract objects."

The difference here is not in *what* is reported, but rather in *how* it is reported. This all suggest that *if* DISTRIBUTION is true (has all true instances), then the following should be true as well:

> ARBITRARY DISTRIBUTION (INFORMAL): If one believes the conjunction of some propositions, then one believes the conjuncts of that conjunction.

I am going to argue, however, that ARBITRARY DISTRIBUTION is false. And so since ARBITRARY DISTRIBUTION is true if DISTRIBUTION is, DISTRIBUTION is false as well (has false instances).

In order to formulate the argument against ARBITRARY DISTRIBUTION I have in mind, we are going to need a more expressive language that allows for quantification over both propositions and pluralities of propositions. Let $\mathcal{L}$ be a language with a countable collection of propositional variables $p, q, r \ldots$ and a countable collection of *plural* propositional variables $pp, qq, rr, \ldots$. Suppose that $\mathcal{L}$ contains, in addition to all the Boolean constants, a belief operator B, and our operator $\equiv$ expressing propositional identity, the following primitive operators: a quantifier $\forall$ that binds propositional variables; a quantifier, also written $\forall$, that binds plural propositional variables; an operator $\prec$ that combines with a formula φ and a plural propositional variable pp, to form a formula $\varphi \prec pp$; an operator $\bigwedge$ that combines with a plural propositional variable pp to form a formula $\bigwedge pp$.

Intuitively $\bigwedge pp$ will be our way of expressing "the conjuncton of the propositions pp," and $\varphi \prec pp$ is our way of stating "the proposition that φ is one of the propositions pp." I will use standard abbreviations in this language, taking, e.g., $\exists p\varphi$ to abbreviate $\neg\forall p\neg\varphi$.

With these notions in place, ARBITRARY DISTRIBUTION can now be expressed more formally as follows:

> ARBITRARY DISTRIBUTION: $\forall pp(B \bigwedge pp \rightarrow \forall p(p \prec pp \rightarrow Bp)$

That is, for any propositions pp, if one believes the conjunction of pp, then one believes every proposition in pp (so we are taking a "conjunct" of $\bigwedge pp$ to simply be one of pp).

My argument against ARBITRARY DISTRIBUTION will depend on a principle that seems to me to be quite plausible from the perspective of a fine-grained theory of propositions. Now from *my* perspective, the perspective of a Boolean, the principle is not very plausible. So the particular argument I am going to give is not one that I think is sound, but rather one that I think an advocate of the Conflation Argument should think is sound. This provides them with their own reasons to reject DISTRIBUTION, even if those reasons are not necessarily my own.

Okay so before stating the principle we'll need a couple of abbreviations. Say that a proposition p if *non-conjunctive*, $NC(p)$, if there are no propositions pp such that $p \equiv \bigwedge pp$. In other words, we'll use $NC(p)$ as an abbreviation for $\neg\exists pp(p \equiv \bigwedge pp)$. We say that *some* propositions pp are *non-conjunctive*, $NC(pp)$, if every one of pp is non-conjunctive. That is, we'll use $NC(pp)$ as an abbreviation for $\forall p(p \prec pp \to NC(p))$. Then the principle that I think the Structured theorist will want to accept is the following:

Non-Conjunctive Belief: $NC(B\varphi)$

Non-Conjunctive Belief is a schema. Intuitively, it says that the proposition that one believes that φ is not itself a non-conjunctive proposition; this proposition is not yielded by applying the conjunction operation $\bigwedge$ to some plurality of propositions pp. Suppose this were false. Then there would be some proposition p that was both the result of applying the belief operator B to a proposition q, *and* the result of applying the conjunction operator $\bigwedge$ to some propositions qq. But this seems like exactly the sort of claim the structured view of propositions is designed to rule out. So, it seems to me, any structured proposition theorist should accept Non-Conjunctive Belief.

There is a principle, however, that I think the structured theorist should accept, even though it is superficially inconsistent with their view. And this is just the principle that conjunctions like $\varphi \wedge \psi$ shouldn't themselves be non-conjunctive:

Non-Conjunctive Harmony: $\neg NC(\varphi \wedge \psi)$

This implies that $\varphi \wedge \psi$ is both the result of applying $\wedge$ to the pair of propositions φ and ψ *and* the result of applying $\bigwedge$ to some plurality of propositions. But given the tight connection between belief reports like 'Alices believes the conjunction of Logicism and Platonism' and 'Alice believes that … and …' it seems to me natural to hold that $\varphi \wedge \psi \equiv \bigwedge pp$ where pp is the plurality consisting of just φ and ψ. Now strictly speaking this principle is indispensable to my argument so I won't belabor its defense. But it simplifies things a bit so I'll assume it.

With these assumptions on the table, the problem is now the following. Given extremely plausible, broadly logical, background assumptions, one can show that from Non-Conjunctive Belief the following uniqueness claim is *false*:

Uniqueness: $\forall pp \forall qq((NC(pp) \wedge NC(qq) \to (B \bigwedge pp \equiv B \bigwedge qq) \to pp \equiv qq))$

That is, it can be shown that for some non-conjunctive propositions pp and qq, to believe the conjunction of pp just is to believe the conjunction of qq, despite the fact pp are not qq. In the Appendix to this paper the proof is presented. But the argument is essentially Cantorian. Here I will sketch out the reasoning informally. According to Uniqueness, the "operator" $B \bigwedge(\cdot)$ yields distinct propositions when applied to distinct pluralities of non-conjunctive propositions. And according to Non-Conjunctive Belief, the propositions that $B \bigwedge(\cdot)$ yields when applied to some non-conjunctive propositions is itself a non-conjunctive proposition. So Non-Conjunctive Belief together with Uniqueness require that $B \bigwedge(\cdot)$

constitute an "injection" from pluralities of non-conjunctive propositions to non-conjunctive propositions. By a plural analogue of Cantor's theorem, there can be no such injection.

Now this talk of "injections" and "operations" is all intended here to be loose speech. The result can be proven within the object language using only logical assumptions. The above sketch of the reasons is merely intended to illustrate the basic idea. The important point is the bearing of the failure of UNIQUENESS on ARBITRARY DISTRIBUTION. Since UNIQUENESS is false, there are some non-conjunctive propositions pp and qq such that $pp \not\equiv qq$ despite the fact that $B \bigwedge pp \equiv B \bigwedge qq$. Now suppose, without any loss of generality, that there is some $q \prec qq$ such that $q \not\prec pp$. And suppose that one comes to believe that $\bigwedge pp$ by considering each proposition $p \prec pp$ individually, and then infers that $\bigwedge pp$. Then we have $B \bigwedge pp$. And so since $B \bigwedge pp \equiv B \bigwedge qq$ it follows by LEIBNIZ'S LAW that $B \bigwedge qq$. Then, by ARBITRARY DISTRIBUTION, it follows that $\forall q(q \prec qq \rightarrow Bq)$: that is, one believes every proposition q that is one of qq. But since some $q \not\prec pp$, there is some q that is not one of the propositions that explicitly considered in coming to form the belief that $\bigwedge pp$. Moreover, and here is where the restriction to non-conjunctive propositions becomes important, none of pp are conjunctive propositions. So one cannot have acquired this belief in q by believing some conjunctive proposition in pp one of whose conjuncts is q; by assumption there are no such conjunctions. Thus it would appear that even for the structured theorist, ARBITRARY DISTRIBUTION places a closure condition on belief that, on its face, is not acceptable. Given ARBITRARY DISTRIBUTION, one seems to almost as if by magic acquire this belief in q, a proposition that is neither one explicitly considered in deciding that $\bigwedge pp$, nor a conjunct in a proposition one explicitly considered in coming to believe $\bigwedge pp$. But this seems like exactly the sort of considerations that the structure theorist levels against the Boolean as a reason to give up their view. Thus if they find such considerations convincing enough to reject BOOLEANISM, they should find them convincing enough to reject ARBITRARY DISTRIBUTION, and hence to reject DISTRIBUTION as well.

4. CONCLUSION

If DISTRIBUTION is false, as I have argued, why, then, does it *seem* true? A full theory should tell us. Here, I will merely put forward an hypothesis that I find plausible. A defense of the hypothesis will have to wait. Though I will say a few things in its favor.

Here is the basic thought. If our intuition that DISTRIBUTION holds is not based off any insight into the logic of belief, then a plausible alternative place from which it may arise are the methods we use for assessing conjunctive *sentences*. Consider the following rule:

> ACCEPTANCE DISTRIBUTION: Accept as true $\ulcorner \varphi \wedge \psi \urcorner$ only if you accept as true φ and accept as true ψ.

Suppose this rule obtains the status of a convention. It is common knowledge that speakers follow this rule: thus when a speaker accepts $\ulcorner \varphi \wedge \psi \urcorner$, we *expect* them to accept φ and accept ψ. Now combine this fact with the following test for belief:

> Belief Test: To test whether a competent speaker of the language satisfies $\ulcorner x$ believes that $\varphi \urcorner$, see whether they accept φ.

This is of course not held to be a law, or anything like that. It is rather a heuristic we use to figure out what our peers believe.[24] But together, these two principles predict that *in general*, speakers will expect Distribution to hold. For suppose I want to know whether you believe that $\varphi \wedge \psi$. Then if I employ the Belief Test, I will first try to assess whether you accept '$\varphi \wedge \psi$'; then, because I expect Acceptance Distribution to hold in general, if I decide that you believe that $\varphi \wedge \psi$ on the basis of the Belief Test, I will also conclude, on the basis of Disquotation, that you believe that φ and that you believe that ψ.

This shows that there are a variety of heuristics we plausibly follow that independently predict our intuition that Distribution holds. Given the controversial consequences Distribution can be seen to have for systematic theorizing about propositional attitudes, it seems to me a better working hypothesis that some suitably strong, and simple, theory of fineness of grain is true, like, Booleanism, while our intuitions to the contrary are explained by these slightly less elegant heuristics.

Appendix

Let $\mathcal{L}$ be the following formal language:

$$\varphi \coloneqq p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid B\varphi \mid \varphi \equiv \varphi \mid \forall p_i \varphi \mid \forall pp_i \varphi \mid \varphi \prec pp \mid \bigwedge pp$$

Definition 4.1. Let $\vdash$ be the proof system in $\mathcal{L}$ given by the following schemas and rules:

Taut: all propositional tautologies — MP: $\varphi, \varphi \to \psi / \psi$

UI: $\forall x\varphi \to \varphi[t/x]$ — UG: $\varphi \to \psi / \varphi \to \forall x \psi$

LL: $\varphi \equiv \psi \to (\chi \to \chi[\varphi/\psi])$ — ID: $\varphi \equiv \varphi$

Comp: $\exists pp \forall q(q \prec pp \leftrightarrow \varphi)$ (pp not free in φ)

Proposition 4.2. Uniqueness and Non-Conjunctive Belief are jointly inconsistent in $\vdash$.

Proof. Let p be an arbitrary variable and consider the formula

$$\chi(p) \coloneqq \neg\forall qq((\mathsf{NC}(qq) \wedge B \bigwedge qq \equiv p) \to p \prec qq)$$

[24] In this respect the approach I favor has more in common with Saul (2007) and Williamson (2020) than traditional Gricean theories in accounting for recalcitrant semantic intuitions. The hypothesis is that such intuitions stem from common psychological processes we use in assessing the truth of various sentences.

From Comp there are some rr such that

$$q \prec rr \leftrightarrow \chi(q)$$

Assume for contradiction that $\neg\chi(B \bigwedge rr)$. Then

$$\forall qq((\mathsf{NC}(qq) \wedge B \bigwedge qq \equiv B \bigwedge rr) \rightarrow B \bigwedge rr \prec qq)$$

By ID we have $B \bigwedge rr \equiv B \bigwedge rr$ and by Non-Conjunctive Belief we have $NC(B \bigwedge rr)$. Thus $B \bigwedge rr \prec rr$ and so $\chi(B \bigwedge rr)$, contradicting the assumption. Thus $\chi(B \bigwedge rr)$. So there are some qq such that $NC(qq)$ and $B \bigwedge qq \equiv B \bigwedge rr$ and $B \bigwedge qq \not\prec qq$. By LL, $B \bigwedge rr \not\prec qq$. But since $\chi(B \bigwedge rr)$, $B \bigwedge rr \prec rr$. This contradicts Non-Conjunctive Belief. □

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